Average treatment effect

Checking ATE assumptions

Assessing treatment effect using registry data

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Summary

Motivation

Objective: decide whether a treatment is beneficial \rightarrow for a give time horizon

1 year

 $M_{\rm ATERIAL:}$ registry data

- observational data (i.e. non-randomized)
- Iong term follow-up
- large number of patients

What do we mean by beneficial:

• does the treatment reduce the 1-year risk of developing the disease ?

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Plan



- Estimating a 1-year risk of a disease using registry data
- 2 Estimating a treatment effect using registry data
 - e.g. Staerk et al. 2016:



Stroke/thromboembolism

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Plan

- Isstimating a 1-year risk of a disease using registry data → model checking [Cox]
- ② Estimating a treatment effect using registry data → model checking [new strategy]
 - e.g. Staerk et al. 2016:



Stroke/thromboembolism

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Summary

Absolute risk

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Definition

1-year absolute risk:

- chance that a person will be diagnosed with the event in 1 year
 - \rightarrow depends on the risk of the event λ_{event}
 - \rightarrow depends on the risk of death λ_{death}







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Summary

Definition

1-year absolute risk:

- chance that a person will be diagnosed with the event in 1 year
 - \rightarrow depends on the risk of the event $\lambda_{\textit{event}}$



X: covariates like age, gender ...

Considering registry data, are involved:

- the event of interest
- competing risks, e.g. death
 - \rightarrow will prevent the observation of the event

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Summary

Definition

1-year absolute risk:

- chance that a person will be diagnosed with the event in 1 year
 - \rightarrow depends on the risk of the event $\lambda_{\textit{event}}$
 - \rightarrow depends on the risk of death λ_{death}



X: covariates like age, gender \dots

Considering registry data, are involved:

- the event of interest
- competing risks, e.g. death
 - \rightarrow will prevent the observation of the event

Summar

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Definition

1-year absolute risk:

Absolute risk

- chance that a person will be diagnosed with the event in 1 year
 - \rightarrow depends on the risk of the event $\lambda_{\textit{event}}$
 - ightarrow depends on the risk of death $\lambda_{\textit{death}}$

$$r_{event}(t|X) = \underbrace{\int_{0}^{t}}_{\text{addition over time}} \underbrace{\underbrace{S_{0}(s|X)}}_{\text{survival at to time s immediate risk of the event at time s}}_{immediate risk of the event at time s} ds$$

X: covariates like age, gender . . .

Considering registry data, are involved:

- the event of interest
- competing risks, e.g. death
 - \rightarrow will prevent the observation of the event

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Summary

Cause specific Cox model

One Cox regression for each competing risk:

$$\lambda_{event}(t|X) = \lambda_{0,event}(t) \exp(X\beta_{event})$$
$$\lambda_{death}(t|X) = \lambda_{0,death}(t) \exp(X\beta_{death})$$

We can then estimate the overall survival.

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Summary

Cause specific Cox model

One Cox regression for each competing risk:

$$\lambda_{event}(t|X) = \lambda_{0,event}(t) \exp(X\beta_{event})$$
$$\lambda_{death}(t|X) = \lambda_{0,death}(t) \exp(X\beta_{death})$$

We can then estimate the overall survival.

- no event
- not dead

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Summary

Cause specific Cox model

One Cox regression for each competing risk:

$$\lambda_{event}(t|X) = \lambda_{0,event}(t) \exp(X\beta_{event})$$

 $\lambda_{death}(t|X) = \lambda_{0,death}(t) \exp(X\beta_{death})$

We can then estimate the overall survival.

$$S_0(t|X) = \exp\left(-\int_0^t \lambda_{death}(s|X) + \lambda_{event}(s|X)ds
ight)$$

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Summary



- 1 > library(riskRegression)
- 2 > data(Melanoma)
- 3 > fit1 <- CSC(formula=Hist(time,status)~sex+invasion+age,</pre>
- 4 + data=Melanoma)
- 5 fit1\$models\$`Cause 1`

```
Call:
```

```
survival::coxph(...)
```

	coef	exp(coef)	<pre>se(coef)</pre>	Z	р
sexMale	0.66338	1.94135	0.26632	2.49	0.01274
invasionlevel.1	1.03717	2.82122	0.32824	3.16	0.00158
invasionlevel.2	1.40323	4.06830	0.38074	3.69	0.00023
age	0.00982	1.00987	0.00834	1.18	0.23884

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Summary





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Summary

Summary

We can easily compute the absolute risk

- using one Cox model for the event of interest
- using another Cox model for the competing events

But now we have to check the assumptions for each Cox model !

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Summary

Cox model assumptions

Assumptions:

- proportional hazard (PH) assumption
- 2 (linear) functional form
- (absence of) interaction

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Checking Cox model assumptions

Model checking is more complex compared to a linear regression

- several types of residuals
- many different diagnostic tools
 - validity of the null hypothesis

e.g. PH vs. non PH

- against a specific alternative hypothesis

e.g. quadratic vs. linear effect age

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Checking ATE assumptions

(1) Checking Proportional hazard assumption Cox model:

$$\lambda(t|X) = \lambda_0(t)e^{eta X}$$

Here we assume $\beta \perp t$

• Visual checking with Kaplan Meier



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Checking ATE assumptions

Summary

(1) Checking Proportional hazard assumption Cox model:

 $\lambda(t|X) = \lambda_0(t)e^{\beta X}$

Here we assume $\beta \perp t$

- \bullet Statistical test: (\mathcal{H}_0) the PH assumption holds,
 - i.e. the cumulative score process follows a brownian bridge
- > plot(gof:::cumres(coxph))



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Checking ATE assumptions

(1) Remedies for non proportional hazard Strategy 1: find the problematic variable and the type of time dependency

• Display of the Schoenfeld residuals (Grambsch et al. 1994)



 $\mathbb{E}[r_{ij}] \approx \beta_j(t_i) - \hat{\beta}_j$

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Checking ATE assumptions

(1) Remedies for non proportional hazard Strategy 1: find the problematic variable and the type of time dependency

• Use a Cox model with time varying effects

$$\lambda(t|X) = \lambda_0(t)e^{\beta(t)X}$$

z1



> plot(timereg::timecox(Surv(time,status) \sim z,

Summary

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Checking ATE assumptions

(1) Remedies for non proportional hazard

Strategy 1: find the problematic variable and the type of time dependency $% \left({{{\left[{{{\left[{{{c}} \right]}} \right]}_{{\left[{{{c}} \right]}}}}_{{\left[{{{c}} \right]}}}} \right]} \right)$

Strategy 2: stratification Cox model:

$$\lambda(t|X, treatment) = \lambda_0(t)e^{\beta X + \gamma treatment}$$

Stratified Cox model:

$$\lambda(t|X, \textit{treatment}) = \lambda_{0,\textit{treatment}}(t) e^{eta X}$$

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Summary

(2) Checking the functional form

$$\lambda(t|X,T) = \lambda_0 e^{\beta X}$$

Here we assume the log of the risk increase linearly with X, e.g. with age.

Diagnostic tools:

- Display martingale residuals
- Comparison with model including a quadratic term or spline



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Summary

(3) Checking possible interactions

$$\lambda(t|X, T) = \lambda_0 e^{\beta X + \gamma treatment}$$

Here we assume that the risk increase independently with X and with *treatment*

Diagnostic tools:

- Display martingale residuals
- Comparison with a model with interactions



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Summary

Limits

In practice model validation is tedious:

- large number of tests
 - at least 2 per variables + PH
 - (i.e. linearity and interaction with treatment)
 - competing risks: two Cox models to check
- unclear alternative hypothesis
 - residual plot can be hard to interpret
- large *n* small *p*
 - overpowered tests (Grøn et al. 2016)
 - \rightarrow may detect unimportant deviations to hypothesis

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Summary

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Observational vs. randomized study

Randomized experiment

- eliminates confounding
- balances all risk factors: known AND unknown
- \rightarrow causal interpretation

Observational studies

- \bullet can ONLY account for known and measured risk factors
- ightarrow establish associations

Causal inference theory:

causal interpretation (under hypothesis) in observational studies

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Summary

Counterfactual outcomes

\mathcal{H} ypothetical world



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Summary

Counterfactual outcomes

\mathcal{H} ypothetical world

We can measure for individual *i* at time *t*: $Y_i^{T=1}(t)$, outcome using intervention 1 $Y_i^{T=0}(t)$, outcome using intervention 0

We can estimate

$$Y_i^{T=1}(t) - Y_i^{T=0}(t)$$

the individual causal effect at t



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Summary

Counterfactual outcomes

 \mathcal{R} eal world



Average treatment effect

Checking ATE assumptions

Summary

Counterfactual outcomes

 $\mathcal{R}\mathsf{eal} ~\mathsf{world}$

We only can measure : $Y_i^{T=1}(t)$ OR $Y_i^{T=0}(t)$



We can infer the average causal effect:

$$ACE(t) = \mathbb{E} \begin{bmatrix} Y^{T=1}(t) - Y^{T=0}(t) \end{bmatrix}$$

$$= \mathbb{E} \begin{bmatrix} Y^{T=1}(t) \end{bmatrix} - \mathbb{E} \begin{bmatrix} Y^{T=0}(t) \end{bmatrix}$$

e.g. (no confounder) $\widehat{ACE}(t) = \sum_{i=1}^{n_1} Y_i^{T=1}(t) - \sum_{j=1}^{n_2} Y_j^{T=0}(t)$

Average treatment effect

Checking ATE assumptions

Summary

G formula

\mathcal{R} eal world: confounders



Average treatment effect

Checking ATE assumptions

Summary

G formula

 \mathcal{R} eal world: confounders

Statistical model: $\mathbb{E}[Y|X, T]$

$$ACE(t) = \sum_{i=1}^{n} \mathbb{E}[Y_i(t)|X_i, T=1] \\ -\mathbb{E}[Y_i(t)|X_i, T=0]$$

Here Y(t)|X, T is the absolute risk



Average treatment effect

Checking ATE assumptions

Summary

Workflow (Christiansen et al. 2015)

Define the population of interest Patients with first-time ischemic stroke (n=19223) Exclusion criteria: atrial fibrillation ...

- 2 Define the intervention (T
- Oefine the event of interest (Y)
- Identify the possible competing events (D)
- Identify the possible confounders/pronostic variable (X)
- Define a statistical model for relating Y, T, and X

Average treatment effect

Checking ATE assumptions

Summary

Workflow (Christiansen et al. 2015)

Define the population of interest

2 Define the intervention (T)

e.g.: antiplatelet regimens for secondary stroke prevention $T{=}0{:}$ ASA

- T=1: Clopidogrel
- T=2: ASA+Clopidogrel
- I Define the event of interest (Y)
- Identify the possible competing events (D)
- Identify the possible confounders/pronostic variable (X)
- Define a statistical model for relating Y, T, and X

Average treatment effect

Checking ATE assumptions

Summary

Workflow (Christiansen et al. 2015)

- Define the population of interest
- **2** Define the intervention (T)
- **③** Define the event of interest (Y)
 - e.g.: fatal or non fatal ischemic stroke
- Identify the possible competing events (D)
- Identify the possible confounders/pronostic variable (X)
- Define a statistical model for relating Y, T, and X

Average treatment effect

Checking ATE assumptions

Summary

Workflow (Christiansen et al. 2015)

- Define the population of interest
- **2** Define the intervention (T)
- **O** Define the event of interest (Y)
- Identify the possible competing events (D)
 - e.g.: death not related to a stroke event
- Identify the possible confounders/pronostic variable (X)
- Define a statistical model for relating Y, T, and X

Average treatment effect

Checking ATE assumptions

Summary

Workflow (Christiansen et al. 2015)

- Define the population of interest
- **2** Define the intervention (T)
- **O** Define the event of interest (Y)
- Identify the possible competing events (D)
- Identify the possible confounders/pronostic variable (X) e.g. age, hypertension, ...
- Define a statistical model for relating Y, T, and X

Average treatment effect

Checking ATE assumptions

Summary

Workflow (Christiansen et al. 2015)

- Define the population of interest
- **2** Define the intervention (T)
- **O** Define the event of interest (Y)
- Identify the possible competing events (D)
- **(3)** Identify the possible confounders/pronostic variable (X)
- **(**) Define a statistical model for relating Y, T, and X

A two-cause specific Cox model:

$$\lambda^{Y}(t|X,T) = \lambda_{0}^{Y}e^{\beta^{Y}X+\gamma^{Y}T}$$
$$\lambda^{D}(t|X,T) = \lambda_{0}^{D}e^{\beta^{D}X+\gamma^{D}T}$$

Checking ATE assumptions

Computation of the G-formula - in @

Package: riskRegression https://github.com/tagteam/riskRegression Function: ate

Arguments

- object: outcome model which describes how event risk depends on treatment and covariates
- data
- treatment: name of the treatment variable in data
- times: time points at which to evaluate risks
- cause: the cause of interest
- B: the number of bootstrap replications used to compute the confidence interval.

Average treatment effect

Checking ATE assumptions

Summary

G-formula (software)

No competing risks:

1 > head(dtSurv)

		time	${\tt strokeEvent}$	${\tt Treatment}$	Age	
	1:	4.901849	FALSE	ТО	59.78796	
	2:	4.555159	TRUE	ТО	60.66406	
	3:	6.681136	FALSE	T1	58.76296	
1	> m(Cox <- coxp	oh(Surv(time	,strokeEver	nt)~ Treatment + A	Age,
2	+		data = dt	Surv)		
3						
4	> at	te(mCox, da	ata = dtSurv	, treatment	t = "Treatment",	
5	+	times =	12, $B = 1000$))		

Average treatment effect

Checking ATE assumptions

Summary

G-formula (software)

Competing risks:

1 > head(dtCR)

		time	eventtype	eventtypeNum	Treatment	Age	
	1:	2.9	stroke	1	ТО	58.96060	
	2:	9.3	censoring	0	ТО	59.37469	
	3:	2.0	death	2	ТО	59.36296	
1	> 1	nCSC <	<- CSC(
2	+		list(H	Hist(time,even	nttypeNum)	~ Treatment	+ Age,
3	+ Hist(time,eventtypeNum)~ Age),						
4	+ data = dtCR						
5	+))					
6							
7	> a	ate(m(CSC,data =	dtCR, treatme	ent = "Trea	atment",	
8	+	ti	imes = 12 ,	cause = 1, B	= 1000)		

Checking ATE assumptions

G-formula (software output)

Absolute risk of stroke relapse

	Treatment	meanRisk	meanRiskBoot	lower	upper	n.boot
1:	ТО	0.111	0.111	0.101	0.123	1000
2:	T1	0.080	0.080	0.071	0.090	1000
3:	T2	0.078	0.078	0.073	0.082	1000

Difference in absolute risk of stroke between treatments:

1	Freatment.A 7	<pre>Freatment.B</pre>	time	diff	
1:	T1	ТО	12	0.032	
2:	T2	ТО	12	0.034	
3:	T2	T1	12	0.002	
	diffMeanBoot	diff.lower	diff	upper	n.boot
1:	0.032	0.017		0.046	1000
2:	0.033	0.022		0.046	1000
3:	0.002	0.002		0.013	1000

Average treatment effect

Checking ATE assumptions

Summary

Assumptions

no unmeasured confounders

- positivity
- well-defined intervention
- correctly specified model
 - proportional hazard assumption
 - D> (linear) functional form
 - D> (absence of) interaction

Average treatment effect

Checking ATE assumptions

Summary

Assumptions

- no unmeasured confounders
- positivity
- well-defined intervention
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 - ▷ proportional hazard assumption
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Average treatment effect

Checking ATE assumptions

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Average treatment effect

Checking ATE assumptions

Assumptions

- no unmeasured confounders
- o positivity
- well-defined intervention
- correctly specified model
 - ▷ proportional hazard assumption
 - ▷ (linear) functional form▷ (absence of) interaction

[Not covered] non-informative censoring, influential observations



 \widehat{ATE} : -0.513[-0.571; -0.441] vs -0.244[-0.281; -0.206]

Average treatment effect

Checking ATE assumptions

Summary

Assumptions

- no unmeasured confounders
- positivity
- well-defined intervention
- correctly specified model
 - ▷ proportional hazard assumption
 - \triangleright (linear) functional form
 - ▷ (absence of) interaction

[Not covered] non-informative censoring, influential observations

 \widehat{ATE} : -0.164[-0.221; -0.107] vs -0.200[-0.264; -0.132]



Average treatment effect

Checking ATE assumptions

Summary

Assumptions

- no unmeasured confounders
- positivity
- well-defined intervention
- correctly specified model
 - ▷ proportional hazard assumption
 - ▷ (linear) functional form
 - ▷ (absence of) interaction





Average treatment effect

Checking ATE assumptions

Summary

Assumptions

- no unmeasured confounders
- positivity
- well-defined intervention
- correctly specified model
 - ▷ proportional hazard assumption
 - ▷ (linear) functional form
 - \triangleright (absence of) interaction

Average treatment effect

Checking ATE assumptions

Summary

Proposed approach

• Compare alternative modelling strategies to the Cox model \rightarrow check result sensitivity to model assumptions

Alternative models:

- increased flexibility:
 - \rightarrow less biased
- ✗ increased complexity:
 - \rightarrow harder to interpret
 - \rightarrow increased variance of the estimates

Average treatment effect

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Summary

Extensions/alternatives to Cox model

• relax PH assumption:

- -Cox strata stratified Cox model
- -Others Cox model with time varying effects logistic risk regression
- include non-linear relationships/interactions
 -Cox spline regression spline in Cox model
 -RF random survival Forest

Average treatment effect

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Summary

Simulation study

Investigate the bias/variance trade-off

Three scenari:

(I) violation of proportional hazard assumption

(II) mispecification of the functional form of a risk factor

(III) missing interaction with the treatment variable

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Results - proportional hazard

Scenario I	Non proportional effect of treatment			
$OR_T(t \leq 5)$	0.8	1.7	7.4	
$OR_T(t \ge 5)$	0.4	0.4	0.4	
ATE	0.181	-0.002	-0.068	
Cox	-0.049 (0.05)	-0.082 (0.083)	-0.062 (0.062)	
Cox strata	0.001 (0.011)	0 (0.01)	-0.01 (0.013)	
Random Forest	0.001 (0.01)	0 (0.009)	0.005 (0.007)	

Table: last three rows:

bias (root mean square error) of the ATE estimated by the models OR_T odd ratio for the treatment effect

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Summary

Results - functional form

Scenario II	Non linear effect of covariate			
OR _T	0.4	0.4	0.4	
OR _{Age}	0.8	2.7	7.4	
ATE	0.338	0.338	0.338	
Cox Cox strata Random Forest	0.001 (0.01) 0.001 (0.011) -0.003 (0.011)	0 (0.01) -0.001 (0.011) -0.004 (0.012)	0 (0.01) -0.001 (0.011) -0.004 (0.012)	

Table: last three rows:

bias (root mean square error) of the ATE estimated by the models OR_T odd ratio for the treatment effect OR_{Age} odd ratio for the non linear effect of the risk factor (increased risk after 50 years)

Average treatment effect

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Summary

Results - interaction

Scenario III	Interaction between treatment and covariate			
ORT	0.4	0.4	0.4	
$OR_{T*gender}$	1.6	2.7	7.4	
ATE	0.075	-0.011	-0.097	
Cox Cox strata Random Forest	-0.01 (0.014) 0.001 (0.011) 0.001 (0.011)	-0.04 (0.041) 0 (0.011) 0 (0.011)	-0.146 (0.146) 0.001 (0.01) 0 (0.01)	

Table: last three rows:

bias (root mean square error) of the ATE estimated by the models OR_T odd ratio for the treatment effect $OR_{T*render}$ odd ratio for interaction between gender and treatment

Average treatment effect

Checking ATE assumptions ○○○○○○● ○○○○○ Summary

Discussion

Alternative models:

- No noticeable increase of the variance of the estimates
- For categorical variables, a fully stratified Cox model is robust against:
 - non PH
 - interaction between variables
- For dealing with continuous variables, use random Forests
 - 🗶 extra-parameters to be tuned (e.g. number of trees)
- ✗ Increased computation time

Average treatment effect

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Application

Objective:

- to compare 3 antiplatelet regimens using the danish registry
- n = 19223 patients
- time horizon: 1 year

Outcome:

• date of first stroke event (n=1610, 8.4%)

Competing event:

• death (n=677, 3.5%)

Possible confounders:

• many (p=10) including age, gender, ...

Average treatment effect

Checking ATE assumptions

Summary

Checking Proportional hazard assumption



¹based on the score process

Average treatment effect

Checking ATE assumptions

Summary

Checking functional form



Variable age

- additional risk after 75 years
- approx. linear

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Summary

Models

Variables:

- Continuous: age
- Categorical: treatment, gender, year

CSC Two cause specific Cox model

CSC inter CSC

- + interactions between treatment and gender, age, year $% \left({{\left({{{\left({{{\left({1 \right)}} \right)}} \right)}_{\rm{c}}}}_{\rm{c}}} \right)} \right)$
- + cubic spline on age
- CSC strata stratified CSC on treatment, gender, year

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Summary

naive Cox model vs. alternatives



 violation of PH assumption impacts the estimate of NTT at early times

Average treatment effect

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Summary

Summary

Cox models can be used to assess :

- ▷ disease incidence (absolute risk)
- ▷ average treatment effect

Relies on several assumptions, e.g.:

- ▷ proportional hazard
- ▷ linear effect, no interaction between variables

Model checking:

- ▷ Hope: no unmeasured confounders
- \triangleright usual diagnostic tools are of limited interest for large p or n
- *proposal*: assess the impact of Cox model assumptions using alternative models

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Checking ATE assumptions

Summary

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